

# Lepton Flavor Changing Charged Current and Interaction

**Ong Jian Fuh and Ithnin Abdul Jalil**

Department of physics, Faculty of Science, University of Malaya, 50630 Kuala Lumpur, Malaysia

E-mail: thoms2543@yahoo.com, isninaj@gmail.com

## **Abstract.**

We write the neutrino mass eigenstate into the weak doublet of the group  $SU(2)_L$ . The massive neutrino is written as the mixture of the flavor neutrino. The flavor changing Lagrangian of lepton-W boson coupling were obtained. The neutrino mixing angle appear in the vertex factor of the interaction of flavor neutrino with charged lepton. The results lead to the Feynman diagram of neutrino oscillation in matter and play an important role in  $(\nu_e - e)$  elastic scattering. The anomalous muon decay  $\mu^- \rightarrow e^- + \bar{\nu}_\mu + \nu_e$  take part in giving the muon decay lifetime of  $2.197019\mu s$ . The branching ratio  $B(\mu^- \rightarrow e^- + \bar{\nu}_\mu + \nu_e)$  is obtained.

PACS numbers: 13.15.+g, 14.60.Pq, 12.15.Ff

## 1. Introduction

A neutral particle was first postulated in 1930 by Pauli in order to describe the missing energy in the nuclear beta decay [1]. This neutral particle was later named by Fermi as neutrino. The neutrinos are produced from the beta decay and propagate into the vacuum at almost the speed of light and pass through the matter at almost no interaction. The nuclear beta decay is described by weak interaction. In particular, only lepton and quark participate in weak interaction. The weak interaction was later unified with electromagnetic interaction by Weinberg, Salam and Glashow to give a more complete theory called electroweak theory [2]. In this model, the neutrino is assumed to be massless. The zero mass of the neutrino makes the model as complete and further improvement is not necessary. In particular, the muon decay and the elastic scattering of electron-neutrino on the electron ( $\nu_e - e$ ) agree to a very high degree of accuracy with the experimental data [3, 4]. However, the studies of the solar neutrino problem [5] have led physicists to suspect that one type of neutrino may transform into another type during propagation from the sun to the earth. The flavor changing or so called neutrino oscillation is later known to be possible if the neutrino is massive. A detailed study on neutrino oscillation was carried out by Belinky and Pontecorvo [6]. In their study, the weak interaction of leptons was constructed by the analogy with the weak interaction of quarks. In the theory of quark, the d, s, b quark flavor eigenstate form a weak doublet with u, c, t quark mass eigenstate. However, the quark do not couple to a definite flavor state in the weak charged current, but to the linear combination of their mass eigenstate d, s, b described by Cabibbo-Kobayashi-Maskawa (CKM) matrix [7, 8]. The same situation was applied to neutrino where the flavor eigenstate of  $\nu_e, \nu_\mu, \nu_\tau$  are in linear combination with the mass eigenstate  $\nu_1, \nu_2, \nu_3$  describe by neutrino mixing matrix [9]. However, in the theory of neutrino the observed state is only the flavor eigenstate, in contrast to the quark where the observed state is their mass eigenstate. As discuss in (9), the quark lepton analogy is not perfect. The mass eigenstate of quark are also flavor eigenstate since the mixing matrix is nearly diagonal. On the other hand, only the neutrino  $\nu_e, \nu_\mu, \nu_\tau$  is flavor eigenstate while the mass eigenstate  $\nu_1, \nu_2, \nu_3$  cannot be associated with any particular lepton flavor. Thus the lepton charged current preserved the flavor symmetry but not to quark weak charge current. In standard model (SM), the lepton numbers are conserved. The electron and its neutrino are assign to a lepton number  $L_e = 1$ , and  $L_e = -1$  to their antiparticle and the same for  $L_\mu$  and  $L_\tau$ . It does not allow the couple of charged lepton with various flavors of neutrino as in charged quark does. Furthermore, the type of neutrino produced from beta decay is determined by the lepton number conservation. However, since the observed neutrino oscillation phenomenon does not follow from the lepton number conservation and so we need not to have any of these restriction. The charge current of lepton should not only couple to definite flavor neutrino. It is the purpose of our work to study the conditions which allow the nonconservation of lepton number in the energy scale of electroweak interaction. In doing this, we should not impose any flavor symmetry of charged lepton and neutrino

field into the doublet of the group  $SU(2)_L$  [10]. We shall get the lepton charged current that allow the flavor changing as quark does. The plan of this paper is as follow. We shall start by reviewing the neutrino mixing in the SM. We begin our work by assuming that the neutrinos enter the left handed doublets which distinguish only in their mass and writing down the charged current. We formulate the new Lagrangian by inserting the neutrino mixing into the theory. We then derive the interaction Lagrangian of neutrino with leptons and gauge bosons. We then show the vertex factors and Feynman diagrams for the modified theory. The study of  $(\nu_e - e)$  elastic scattering and muon decay were carried out to test the model and compare the result to the current SM and experimental result.

## 2. Neutrino Mixing

In the SM, the flavor neutrino enter the left handed doublet of the group  $SU(2)_L$  as

$$\psi_L^l = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad (1)$$

which conserve lepton number. The neutrino field,  $\nu_l$  have acquired mass through Higgs-lepton Yukawa coupling to the right handed neutrino field [11]. Thus, the coupling of gauge bosons to neutrino is given by

$$\begin{aligned} \mathcal{L}_I = & -gj_W^\alpha W^{\alpha\dagger} + H.c + \frac{g}{4\cos\theta_W} \{ \bar{\nu}_e \gamma^\alpha (1 - \gamma^5) \nu_e Z_\alpha + \bar{\nu}_\mu \gamma^\alpha (1 - \gamma^5) \nu_\mu Z_\alpha \} \\ & - \frac{1}{v} \{ m_{\nu_e} \bar{\nu}_e \nu_e \sigma + m_{\nu_\mu} \bar{\nu}_\mu \nu_\mu \sigma \} \end{aligned} \quad (2)$$

and

$$\begin{aligned} j_W^\alpha &= \frac{1}{\sqrt{2}} \sum_l \bar{\nu}_{lL} \gamma^\alpha l_L \\ \nu_{lL} &= \frac{1}{2} (1 - \gamma^5) \nu_{lL} \end{aligned} \quad (3)$$

where  $v = \left( \frac{-\xi^2}{\lambda} \right)^{\frac{1}{2}}$ ,  $\xi^2$  and  $\lambda$  are arbitrary real parameter. However, the Lagrangian can be written in term of neutrino mass eigenstate through neutrino mixing.

$$\nu_l = \sum_{i=1,2} U_{il} \nu_i \quad (4)$$

where

$$U_{il} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad (5)$$

is the neutrino mixing matrix. In terms of mass eigenstate, the Lagrangian takes the form of

$$\begin{aligned}\mathcal{L}_I = & -\frac{g}{\sqrt{2}}[(\bar{\nu}_1, \bar{\nu}_2)_L \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} e \\ \mu \end{pmatrix}_L] W^{\alpha\dagger} + H.c \\ & + \frac{g}{4 \cos \theta_W} \{ \bar{\nu}_1 \gamma^\alpha (1 - \gamma^5) \nu_1 Z_\alpha + \bar{\nu}_2 \gamma^\alpha (1 - \gamma^5) \nu_2 Z_\alpha \} \\ & - \frac{1}{v} \{ m_1 \bar{\nu}_1 \nu_1 \sigma + m_2 \bar{\nu}_2 \nu_2 \sigma - m_{12} (\bar{\nu}_1 \nu_2 + \bar{\nu}_2 \nu_1) \}\end{aligned}\quad (6)$$

where  $m_{12}$  is the mass different between the neutrino mass eigenstate. This result [11] is well known in the neutrino research community.

### 3. Flavor Changing Charged Current

Base on the facts discussed in section 1, we propose a model of lepton

$$\psi_L^l = \begin{pmatrix} \nu_i \\ l \end{pmatrix}_L = \begin{pmatrix} \nu_1 \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_2 \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_3 \\ \tau \end{pmatrix}_L \quad (7)$$

where the neutrino fields are non degenerate in mass as the charged leptons do. As a particle the neutrino is not really massless [11]. The neutrino is assumed to be produced in weak decays with a definite mass. The coupling of the massive neutrino is separated according to their mass categories with charged lepton. Since the mass of the electron is less than muon and in turn the mass of muon is less than that of tauon. So we assume the corresponding massive neutrinos follow this mass category. Therefore, the current that couple leptons to the W boson field become

$$j_W^\alpha = \frac{1}{\sqrt{2}} \sum_i \bar{\nu}_{iL} \gamma^\alpha l_L \quad (8)$$

Let the  $\nu_1, \nu_2, \nu_3$  be written as the linear combination of flavor eigenstate as similar to Equation (4)

$$\begin{aligned}\nu_1 &= \cos \theta \nu_e + \sin \theta \nu_\mu \\ \nu_2 &= -\sin \theta \nu_e + \cos \theta \nu_\mu\end{aligned}\quad (9)$$

In term of neutrino flavor eigenstate, the two components lepton-W boson coupling Lagrangian takes the form of

$$\mathcal{L}_I^{LW} = -\frac{g}{\sqrt{2}}(\bar{\nu}_e, \bar{\nu}_\mu)_L \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} e \\ \mu \end{pmatrix}_L W^{\alpha\dagger} + H.c \quad (10)$$

The same procedure is applied to neutral current and lepton Higgs coupling and one obtain

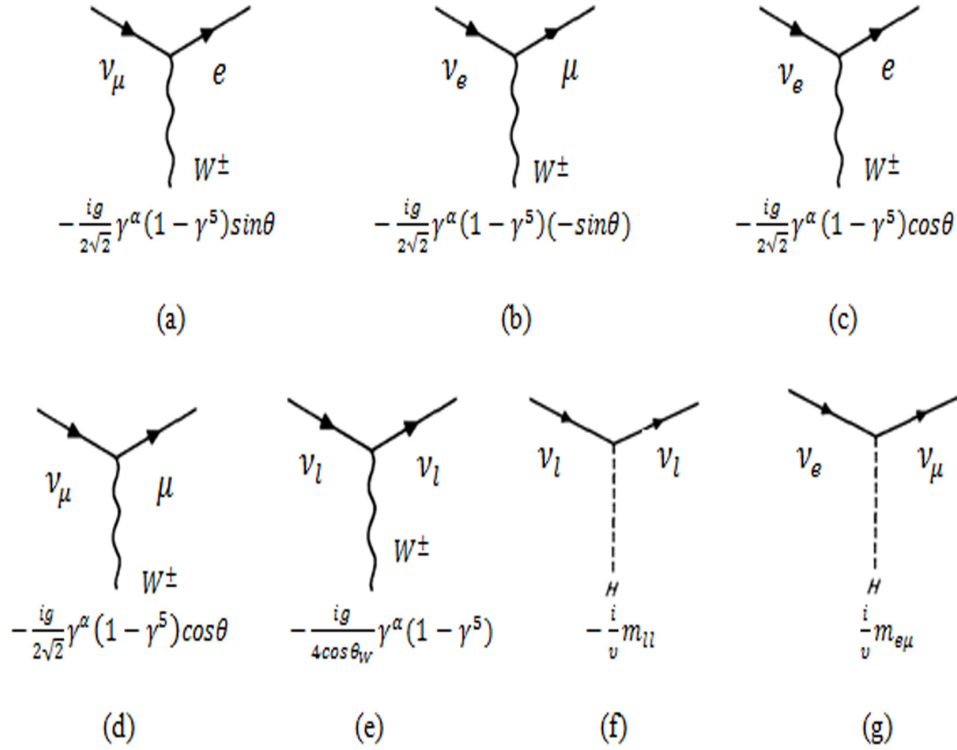
$$\mathcal{L}_I^{LZ} = \frac{g}{4 \cos \theta_W} \{ \bar{\nu}_e \gamma^\alpha (1 - \gamma^5) \nu_e Z_\alpha + \bar{\nu}_\mu \gamma^\alpha (1 - \gamma^5) \nu_\mu Z_\alpha \} \quad (11)$$

and

$$\mathcal{L}_I^{LH} = -\frac{1}{v}\{m_{ee}\bar{\nu}_e\nu_e\sigma + m_{\mu\mu}\bar{\nu}_\mu\nu_\mu\sigma - m_{e\mu}(\bar{\nu}_e\nu_\mu + \bar{\nu}_\mu\nu_e)\} \quad (12)$$

We should emphasis that the field  $\nu_l$  from Equation (10) to (12) is the flavor neutrino field with mass  $m_{ll}$  in contrast to the Equation (6) where the neutrino field is the mass eigenstate with mass  $m_i$ .

After obtaining the Lagrangian of electroweak interaction with neutrino mixing, it is interesting to construct the basic vertices and the Feynman diagrams. Basically there are 14 vertices in the Lagrangian but we will only discuss 7 of them and the rest are the Hermitian conjugate terms. The vertex factors and Feynman diagrams are shown in the following figure.



**Figure 1.** The vertex factor and Feynman diagrams

The Feynman diagrams in Figure 1 allow the possibility for the interaction of charged lepton with different flavor of neutrinos. The vertices in Figure 1(a) and 1(b) carries a factor of  $\sin \theta$  and  $-\sin \theta$  respectively while Figure 1(c) and 1(d) carries a factor of  $\cos \theta$  ; apart from that they are identical to the SM. When the mixing angle is small, the lepton numbers violating vertex factors are suppressed or weaker than the lepton number conserving one. However, the vertices in Figure 1(e) and 1(f) remain the same as in SM and no mixing parameter is involved. The term in Figure 1(g) shows that the neutrino change their flavor when interact with the Higgs boson. The vertex factors

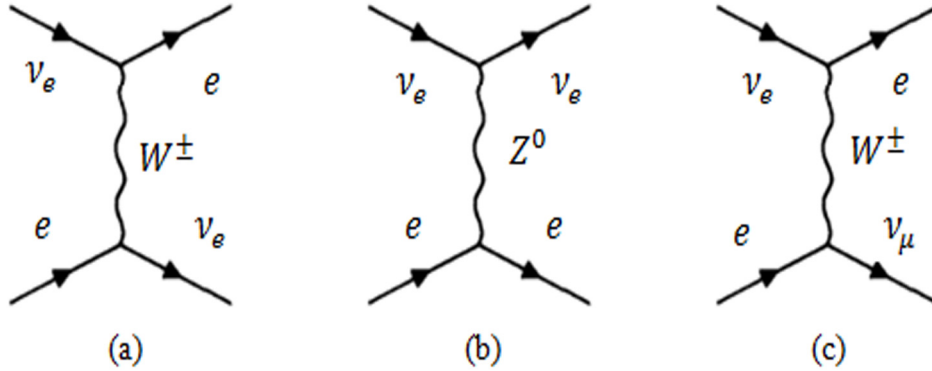
obtained above is the same to the vertex factor of charged weak interaction of quarks [12] which was suggested by Cabibbo where the later one involves Cabibbo angle [7] .

#### 4. Neutrino Interaction

After obtaining the vertex factor from the Lagrangian, it is desirable to study the interaction of neutrino with other particles where the mixing angle is taken into consideration. In fact, some interaction that is forbidden in SM appears to be possible. The properties of neutrino oscillation would become apparent and give a deeper understanding for the electroweak interaction

##### 4.1. Electron- Neutrino Electron ( $\nu_e - e$ ) Elastic Scattering

As our first application of the modified theory of neutrino, we would like to consider the ( $\nu_e - e$ ) elastic scattering. This process proceeds through charge current (CC), Figure 2(a) and neutral current (NC), Figure 2(b) channels in the SM. Nevertheless, if neutrino mixing is included, the CC and NC channels and their interference would not be enough to contribute to the scattering cross section as measured in experiment. Hence, there would be other process that would take part in the scattering. Therefore, we would like to introduce the new process as illustrate in Figure 2(c) as part of the scattering process.



**Figure 2.** Feynman diagram for ( $\nu_e - e$ ) elastic scattering

In Figure 2(c) the process can be interpreted as neutrino oscillation in matter [13] where the electron neutrino changes into muon neutrino after scattering with electron.

$$\nu_e + e^- \rightarrow \nu_\mu + e^- \quad (13)$$

The explicit cross section for the ( $\nu_e - e$ ) elastic scattering process shown in Figure 2 can be written as

$$\sigma^{\nu_e} = \sigma^{CC} + \sigma^{NC} + \sigma^I + \sigma^{\nu_e \rightarrow \nu_\mu} \quad (14)$$

where  $\sigma^I$  is the interference term of CC and NC channels whereas  $\sigma^{\nu_e \rightarrow \nu_\mu}$  do not interfere with other channels since the final state is different. The amplitudes contributed from the diagram of Figure 2 is

$$\begin{aligned} M_{CC} &= \frac{-ig^2 \cos^2 \theta}{8(k^2 - m_W^2)} \bar{u}(\vec{p}') \gamma_\beta (1 - \gamma_5) u(\vec{p}) \bar{u}(\vec{q}') \gamma^\beta (1 - \gamma_5) u(\vec{q}) \\ M_{NC} &= \frac{ig^2}{8(k^2 - m_Z^2)} \bar{u}(\vec{p}') \gamma_\beta \left( 2 \sin \theta_W - \frac{1}{2} + \frac{1}{2} \gamma_5 \right) u(\vec{p}) \bar{u}(\vec{q}') \gamma^\beta (1 - \gamma_5) u(\vec{q}) \\ M_{\nu_e \rightarrow \nu_\mu} &= \frac{-ig^2 \cos \theta \sin \theta}{8(k^2 - m_W^2)} \bar{u}(\vec{p}') \gamma_\beta (1 - \gamma_5) u(\vec{p}) \bar{u}(\vec{q}') \gamma^\beta (1 - \gamma_5) u(\vec{q}) \end{aligned} \quad (15)$$

We obtain the expression for the cross section of  $(\nu_e - e)$  elastic scattering at  $k \ll m_W$  is

$$\sigma^{\nu_e} = \frac{G_F^2 s}{\pi} \left[ \cos^4 \theta + \frac{(2 \sin^2 \theta_W - 1)^2}{4} + \frac{\cos^2 \theta (2 \sin^2 \theta_W - 1)}{2} + \frac{\sin^2 2\theta}{4} \right] \quad (16)$$

where  $s = meE_\nu$  is the total energy in the laboratory frame. The first three terms in Equation (16) is due to CC, NC and interference respectively while the last term is due to neutrino flavor changing process. By inserting the values of the weak mixing angle and neutrino mixing angle of solar neutrino from recent available published data [14].

$$\begin{aligned} \sin^2 \theta_W &= 0.23119 \\ \sin^2 2\theta &= 0.86 \end{aligned} \quad (17)$$

one obtain the cross section formula

$$\sigma^{\nu_e} = 9.892 \times E_{\nu_e} (MeV) \times 10^{-45} cm^2 \quad (18)$$

However, the cross section formula in SM obtained by using the same weak mixing angle parameter is

$$\sigma_{SM}^{\nu_e} = 9.524 \times E_{\nu_e} (MeV) \times 10^{-45} cm^2 \quad (19)$$

Meanwhile, the cross section of the  $(\nu_e - e)$  elastic scattering measured by the Liquid Scintillator Neutrino Detector (LSND) using a  $\mu^+$  decay-at-rest  $\nu_e$  beam at the Los Alamos Neutron Science Center gives [14]

$$\sigma_{exp}^{\nu_e} = 10.1 \pm 1.1(stat) \pm 1.0(syst) \times E_{\nu_e} (MeV) \times 10^{-45} cm^2 \quad (20)$$

The accuracy of the cross section we obtain is 2.06% compare to SM which is 5.70%. The cross section we obtain is closer to the experimentally measured value as compare to the SM. The cross section in Equation (16) not only contains the information of CC and NC channels but also contain the information regarding neutrino oscillation in matter.

#### 4.2. Muon Decay

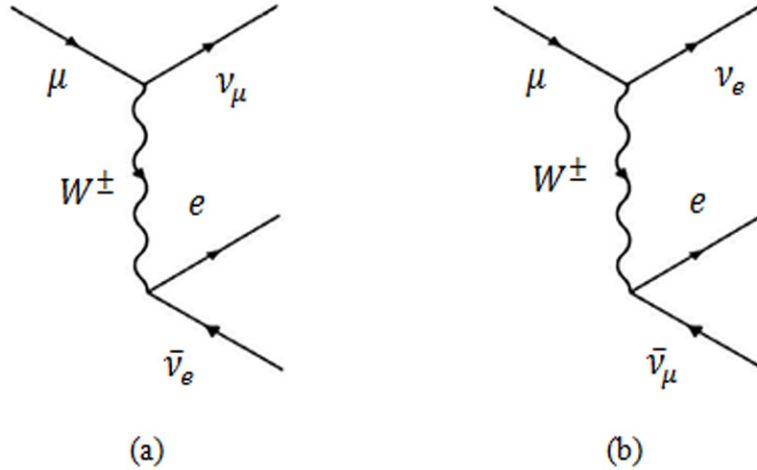
We now move on to the muon decay process where the muon would decay into electron and follow by producing two types of neutrinos. The measurement of the muon decay lifetime in pass few decades have given a precise test to the weak interaction. This muon decay mode,  $\mu^- \rightarrow e^- + \bar{\nu} + \nu$  have the measured lifetime of [14]

$$\tau_\mu = (2.197019 \pm 0.000021) \times 10^{-6} s \quad (21)$$

The measurements of muon lifetime proceed through the detection of electron as it exhibit electromagnetic interaction. However, as to what type of neutrinos it decays into, we do not really certain. It is not even obvious that this neutrino is different from its antiparticle and is still an open question in neutrino research. The studies of lepton flavor changing charged current have led us to suspect that the anomalous muon decay of the following type,

$$\mu^- \rightarrow e^- + \bar{\nu}_\mu + \nu_e \quad (22)$$

would contribute to the muon lifetime of  $2.197019 \mu s$ . Note that the conventional muon decay is  $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ .



**Figure 3.** Muon decays (a)Conventional muon decay ; (b)Anomalous muon decay

The amplitude of the muon decay is written as the sum of both diagrams in Figure 3

$$\begin{aligned} M &= M_a + M_b \\ M_a &= \frac{-ig^2 \cos^2 \theta}{8(k^2 - m_W^2)} \bar{u}(\vec{p}') \gamma^\beta (1 - \gamma_5) v(\vec{q}_1) \bar{u}(\vec{q}_2') \gamma_\beta (1 - \gamma_5) u(\vec{p}) \\ M_b &= \frac{-ig^2 \sin^2 \theta}{8(k^2 - m_W^2)} \bar{u}(\vec{p}') \gamma^\beta (1 - \gamma_5) v(\vec{q}_2) \bar{u}(\vec{q}_1') \gamma_\beta (1 - \gamma_5) u(\vec{p}) \end{aligned} \quad (23)$$

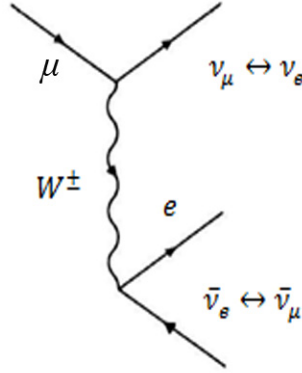
where  $\vec{p}'$ ,  $\vec{q}_1$ ,  $\vec{q}_2$ , and  $\vec{p}$  are momentum for electron, electron type neutrinos, muon type neutrinos and muon respectively. Since the mass of neutrino is extremely small, we may assume that  $\vec{q}_1 = \vec{q}_2$ . The resulting amplitude is

$$M = \frac{-ig^2}{8(k^2 - m_W^2)} \bar{u}(\vec{p}') \gamma^\beta (1 - \gamma_5) v(\vec{q}_1) \bar{u}(\vec{q}_2) \gamma_\beta (1 - \gamma_5) u(\vec{p}) \quad (24)$$

which is just the same as the amplitude given by SM. The amplitude in Equation (24) would give the muon decay lifetime as in Equation (21). The equivalent way in representing the diagrams of muon decay is shown in Figure 4. The neutrino is produced in either muon or electron type and undergo flavor oscillation after that. The branching ratio  $B(\mu^- \rightarrow e^- + \bar{\nu}_\mu + \nu_e)$  is

$$\begin{aligned} B(\mu^- \rightarrow e^- + \bar{\nu}_\mu + \nu_e) &= \frac{\Gamma(\mu^- \rightarrow e^- + \bar{\nu}_\mu + \nu_e)}{\Gamma(\mu^- \rightarrow e^- + \bar{\nu} + \nu)} \\ &= \frac{\Gamma(\mu^- \rightarrow e^- + \bar{\nu} + \nu)}{\Gamma(\mu^- \rightarrow e^- + \bar{\nu} + \nu)} \sin^4 \theta \\ &= 0.098 \end{aligned} \quad (25)$$

which is slightly higher than the experimental upper bound 0.012 [14] for  $\sin^2 2\theta = 0.86$ .



**Figure 4.** Equivalent representation of muon decay

## 5. Conclusion

In this work, the massive neutrinos are classified into their corresponding leptons according to their mass category. Hence, the mass eigenstate is a mixture of flavor eigenstate. In this case, the charged lepton flavor state does not couple to definite neutrino flavor state in the charged current interaction. As a result, the Lagrangian allows the possibility of the lepton number violation interaction. The vertex factors we obtained show the similarity of the weak interaction for quarks and for leptons. The change of neutrino flavor during the interaction with electron can be interpreted as the neutrino oscillation in matter.

and play an important role in the  $(\nu_e - e)$  elastic scattering. The modified theory also shows that the muon decay lifetime is contributed by two types of decay possibility. The probabilities obtained are in good agreement with the experimental data.

## Acknowledgments

The author is grateful for the financial support from the Institute of Postgraduate Studies University of Malaya. This work was supported in part by the Institute of Research Management and Monitoring University of Malaya, Grant No. PS310/2009C.

## References

- [1] W. Pauli *Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol.* **14** ed. K. Winter (Cambridge University Press, United Kingdom, 2000) 1-22
- [2] F. Mandl and G. Shaw *Quantum Field Theory* United Kingdom: John Wiley and Son (1986).
- [3] M. Deniz *et al Phys. Rev. D* **81** 072001 (2010).
- [4] L. B. Auerbach *et al Phys. Rev. D* **63** 112001 (2010).
- [5] J. N. Bachall and R. Davis *Science* **191** 246 (1976).
- [6] S. M. Belinky and B. Pontecorvo *Phys. Rept.* **41** 225 (1978).
- [7] N. Cabibbo *Phys. Rev. Lett.* **10** 531 (1963).
- [8] M. Kobayashi and T. Maskawa *Prog. Theor. Phys* **49** 652 (1973).
- [9] R. D. McKeown and P. Vogel *Phys. Rept.* **394** 315 (2004).
- [10] S. Weinberg *Phys. Rev. Lett.* **19** 1264 (1967).
- [11] R. N. Mohapatra and P. B. Pal *Massive neutrino in Physics and Astrophysics* 3rd ed, World Scientific, (2004).
- [12] D. Griffiths *Introduction to Elementary Particles* John Wiley and Son, (2008), p. 317-319.
- [13] L. Wolfenstein *Phys. Rev. D* **17** 23691 (1978).
- [14] W-M Yao *et al J. Phys. G* **33** 472 (2006).